CALCULATING DRAG AND HEAT TRANSFER IN VISCOUS, QUASISTABLE PIPE FLOW OF SUPERCRITICAL HELIUM

V. M. Eroshenko, V. I. Lubanovskii,
and L. A. Yaskin

UDC 536.24

An integral method allowing for thermal acceleration of the flow is used to obtain data on drag and heat transfer in the laminar flow of helium at supercritical pressure in a uniformly heated circular pipe.

The problem of heat transfer in the flow of supercritical helium in a heated channel of constant cross section models thermal and hydrodynamic processes in the heat-exchanging through parts of cryogenic power plants. Experimental and theoretical studies to data have essentially been limited to the case of turbulent flow, and the literature already contains practical recommendations on calculating heat transfer in this instance [1]. Nevertheless, the possibility of cryostatting systems by laminar convection of the cryogen should not be excluded. The present work reports generalized results of analytical calculation of drag and heat transfer in the viscous flow of supercritical helium in a circular tube at a constant heat flux on the wall.

The system of continuity, motion, and energy equations was solved for the region of quasistable heat transfer using an integral method which is a modification of the calculating scheme of Petukhov and Popov [2]. The principal modification is allowance for thermal acceleration of the flow within the framework of a unidimensional approach. Here, we use the hypothesis of similitude of the axial velocity profile, such as was employed in [3-5], for example.

The motion and energy equations have the form [2]

$$\rho u \frac{\partial u}{\partial x} = -\frac{dp}{dx} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau), \tag{1}$$

$$\rho u \frac{\partial h}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}(rq), \tag{2}$$

where $\tau = -\mu \frac{\partial u}{\partial r}$ and $q = \frac{\lambda}{c_p}\frac{\partial h}{\partial r}$.

To approximately allow for the acceleration and exclude pressure from the variables, we will use the assumption of similitude of the profile of u in the form (see [3], for example)

$$\frac{\partial}{\partial x}\left(\frac{u}{\bar{u}}\right) = 0 \quad \text{or} \quad \frac{\partial u}{\partial x} = \frac{u}{\bar{u}}\frac{d\bar{u}}{dx}, \tag{3}$$

where $\bar{u} = \frac{2}{r_0^2}\int_0^{r_0} urdr$ is the mean velocity in a cross section.

Substitution of Eq. (3) into (1), multiplication of the terms of the latter by rdr, and subsequent integration over the radius from 0 to r_0 gives

$$\frac{dp}{dx} = -\frac{2}{r_0^2}\frac{1}{\bar{u}}\frac{d\bar{u}}{dx}\int_0^{r_0} \rho u^2 rdr - \frac{2}{r_0}\tau_w, \tag{4}$$

where τ_w is the friction stress on the wall.

Having expressed the terms of Eq. (1) with the acceleration and pressure gradient through (3) and (4), respectively, and having integrated from 0 to r, we find the radial distribution of the shear stress:

G. M. Krzhizhanovskii Power Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 3, pp. 373-379, March, 1983. Original article submitted November 12, 1981.

 0022-0841/83/4403-0250$07.50

$$\frac{\tau}{\tau_w} = \frac{r}{r_0} + \frac{1}{r\tau_w}\frac{1}{\bar{u}}\frac{d\bar{u}}{dx}\left[\left(\frac{r}{r_0}\right)^2 \int_0^{r_0} \rho u^2 r dr - \int_0^r \rho u^2 r dr\right]. \quad (5)$$

Now, having replaced τ in the left side of Eq. (5) by $-\mu \frac{\partial u}{\partial r}$ and having integrated the resulting equation from r to r_0, we can derive an implicit expression for velocity:

$$u = \int_r^{r_0} \mu^{-1}\left\{\tau_w \frac{r}{r_0} + \frac{1}{r}\frac{1}{\bar{u}}\frac{d\bar{u}}{dx}\left[\left(\frac{r}{r_0}\right)^2 \int_0^{r_0} \rho u^2 r dr - \int_0^r \rho u^2 r dr\right]\right\} dr. \quad (6)$$

Let us transform the term $\frac{1}{\bar{u}}\frac{d\bar{u}}{dx}$ in (6), allowing for acceleration of the flow. As our calculations showed, it can be assumed with a high degree of accuracy that

$$\overline{\rho u} = \rho_l \bar{u} = \text{const}, \quad (7)$$

where ρ_l is the density calculated from the mean mass temperature T_l. Differentiation of Eq. (7) and subsequent transformations give

$$\frac{1}{\bar{u}}\frac{d\bar{u}}{dx} = -\frac{1}{\rho_l}\frac{d\rho_l}{dx} = -\frac{1}{\rho_l}\frac{d\rho_l}{dT_l}\frac{dT_l}{dx} \simeq \frac{2q_w\beta_l}{\overline{\rho u}c_{pl}r_0}, \quad (8)$$

where β_l and c_{pl} is the coefficient of cubical expansion and the isobaric specific heat, respectively, calculated from T_l. The accuracy of approximation (8) is determined mainly by the contribution of the Joule–Thomson effect to the increase in temperature along the tube axis — which, according to estimates, is negligibly small.

Changing over to the dimensionless coordinate $Y = 1 - r/r_0$ and allowing for (8), we can represent Eq. (6) in the form

$$u = \int_0^Y \left\{\frac{\tau_w r_0(1-Y)}{\mu} + \frac{A}{\mu(1-Y)}[(1-Y)^2\Psi(0) - \Psi(Y)]\right\} dY, \quad (9)$$

where we have used the notation

$$A = \frac{2\beta_l q_w r_0}{\overline{\rho u}c_{p_l}}, \quad \Psi(Y) = \int_Y^1 \rho u^2(1-Y)\,dY.$$

To determine the quantity τ_w in Eq. (9), we can use the flow-rate equation $\overline{\rho u} = 2\int_0^1 \rho u(1-Y)\,dY$. Having replaced the quantity u in the integrand by Eq. (9) and solving the resulting equation for τ_w, we find

$$\tau_w = \left[\overline{\rho u} - 2A\int_0^1 \rho(1-Y)\left\{\int_0^Y [(1-Y)^2\Psi(0) - \Psi(Y)]\frac{dY}{\mu(1-Y)}\right\} dY\right] \Big/ \left[2r_0\int_0^1 \rho(1-Y)\left[\int_0^Y \frac{1-Y}{\mu}dY\right] dY\right] \quad (10)$$

When A = 0, i.e., in the absence of acceleration, Eqs. (9) and (10) are similar to those given by the method in [2].

The enthalpy distribution is determined by the same method as in [2]:

$$h = h_l + \frac{2q_w r_0}{\overline{\rho u}}\left[2\int_0^1 \frac{c_p}{\lambda}\frac{\Phi^2(Y)}{1-Y}dY - \int_0^Y \frac{c_p}{\lambda}\frac{\Phi(Y)}{1-Y}dY\right], \quad (11)$$

where $(Y) = \int_Y^1 \rho u(1-Y)\,dY$.

Accordingly, the expression for the Nusselt number has the form [2]

$$\mathrm{Nu} = \frac{2q_w r_0}{\lambda_l(T_w - T_l)} = \left[2\int_0^1 \frac{\frac{\lambda_l}{\lambda}\frac{c_p}{\bar{c}_p}\Phi^2(Y)}{\overline{\rho u}(1-Y)}dY\right]^{-1}, \quad (12)$$

where $\bar{c}_p = (h_w - h_l)/(T_w - T_l)$.

The friction and total-drag coefficients are respectively determined thus:

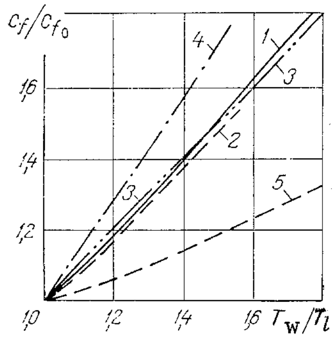

Fig. 1

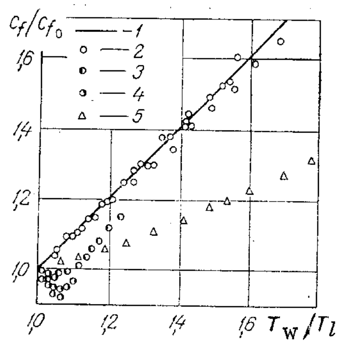

Fig. 2

Fig. 1. Dependence of friction coefficient on temperature factor for helium in the state of an ideal gas: 1) our calculation; 2) [6]; 3) [7]; 4) [8]; 5) calculation at A = 0.

Fig. 2. Dependence of the friction coefficient of supercritical helium on the temperature factor for p = 0.3 MPa: 1) Eq. (15); 2) T_l = 6–15°K; 3) $T_l = T_m$ = 5.6°K; 4) $T_l < T_m$; 5) calculation with A = 0, T_l = 6–15°K.

$$c_f = \frac{2\tau_w \rho_l}{\overline{\rho u^2}}, \tag{13}$$

$$\xi_\Sigma = -\frac{r_0 \rho_l}{\overline{\rho u^2}} \frac{dp}{dx} = c_f + \beta_l (T_w - T_l) \frac{\mathrm{Nu}}{\mathrm{Re\,Pr}} \frac{2\Psi(0)}{\overline{\rho u}\,\overline{u}}. \tag{14}$$

The interaction procedure used in the present work to calculate Nu, c_f, and ξ_Σ from Eqs. (12)-(14), with allowance for (9)-(11), differs from that described in [2] by the fact that the initial data are assigned values of parameters which are usually assumed to be known — specifically, the tube diameter, heat flux on the wall, and the flow rate, pressure, and mean mass temperature of the liquid in the cross section.

To check the above method, we calculated the drag and heat transfer for a theoretically [6, 7] and experimentally [8] investigated case of laminar flow of helium in a tube. The helium was in the state of an ideal gas, when $p = \rho RT$. The calculation was performed for p = 0.1 MPa in the temperature range from 200 to 1500°K. Here, the temperature factor T_w/T_l ranged from 1 to 1.8. In this region of values of the state parameters, c_p = const, and the following approximation holds with a high degree of accuracy for viscosity and thermal conductivity: $\mu/\mu_0 = \lambda/\lambda_0 = (T/T_0)^{0.7}$. The results of calculation of the friction coefficient are shown in Fig. 1. Also shown here are the relations for c_f/c_{f_0} obtained by numerical solution of the complete conservation equations in the boundary-layer approximation [6, 7] (curves 2 and 3) and experimentally [8] (curve 4). It is apparent that our data agrees well with the results of calculations performed by more precise methods, considering both axial and radial convection, and correlates well with measured values of c_f. Calculation by the integral method [2], i.e. without allowance for thermal acceleration, leads to serious underestimation of the friction coefficients (curve 5). The calculated Nusselt numbers show that heat transfer is very slightly dependent on the temperature factor (Table 1), which agrees with the well-known literature data [6-9]. Thus, the proposed modification of the calculating method in [2] makes it possible to correctly predict friction and heat-transfer characteristics in the laminar pipe flow of a monoatomic ideal gas with variable properties. This provides grounds for using the above method to predict drag coefficients and Nusselt numbers for helium in the single-phase, near-critical region of state parameters.

The calculations for supercritical helium embraced the temperature and pressure ranges characteristic of cryogenic power plants currently being designed. The value of T_l was taken equal to $T_m \pm 0.05n$ (where n = 0, 1, 2, ..., 10, where T_m is the pseudocritical temperature at a prescribed pressure) and 6, 7, 8, 9, 10, and 15°K. Pressure ranged from 0.25 (p/p_c = 1.11) to 2.0 MPa. The heat flux on the wall was increased until the temperature on the tube axis for the prescribed T_l was at least 5°K. The properties of helium were calculated from the data in [10]. It was first established that the results for heat transfer and drag cor-

TABLE 1. Dependence of the Nusselt Number on the Temperature Factor for T_l = 700°K, p = 0.1 MPa

T_w/T_l	1,012	1,208	1,407	1,601	1,766
Nu/Nu_0	1,011	1,015	1,008	0,993	0,962

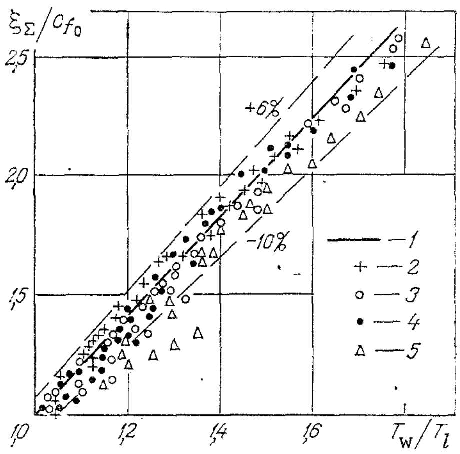

Fig. 3. Generalization of data on total drag: 1) Eq. (16); 2) p = 0.25 MPa; 3) 4; 4) 6; 5) 10.

responding to R = 10^2 and 10^3 were identical, so all of the calculations were performed at a constant Reynolds number of 10^3. The results of calculation of c_f, ξ_Σ, and Nu are shown in Figs. 2-4.

Figure 2 shows the dependence of the ratio of the friction coefficient to its value at constant properties on the temperature factor for p = 0.3 MPa. It is apparent that, at $T_l \lessapprox T_m$, the value of c_f/c_{f_0} can be described by the equation

$$c_f/c_{f_0} = T_w/T_l\ , \tag{15}$$

which to within 1.5% approximates the calculated results for helium in the state of an ideal gas (Fig. 1). At mean mass temperatures lower than T_m, there is a decrease in c_f relative to c_{f_0}, reaching about 10%. The results of calculations without allowance for acceleration for $T_l > T_m$ lie significantly below Eq. (16). The laws reflected by Fig. 2 remain in force at other pressures as well.

Figure 3 shows data on the total drag. Also shown here is the following dependence for an ideal gas

$$\xi_\Sigma/c_{f_0} = 2.1 T_w/T_l - 1.1, \tag{16}$$

which can be derived from Eq. (14) if we consider that $\beta_l = 1/T_l$, $Nu \approx Nu_0 = 4.36$, $c_{f_0} = 16/Re$, and that the quantity $\frac{2\Psi(0)}{\overline{\rho u}\,\overline{u}}$, in accordance with the calculations, is a constant equal to 1.35. It is apparent that this dependence serves as a satisfactory approximation for supercritical helium as well, although here it should be kept in mind that the ratio ξ_Σ/c_{f_0} may be about 20% lower than Eq. (16) at p > 1 MPa and $T_l < T_m$.

The data on heat transfer (Fig. 4) can be generalized with an accuracy sufficient for engineering applications by the following relations:

$$\text{for } \bar{c}_p/c_{p_l} < 1 \quad Nu/Nu_0 = \left(\frac{\rho_l}{\rho_w}\frac{c_{pl}}{\bar{c}_p}\frac{\lambda_l}{\lambda_w}\right)^{-0.3}, \tag{17}$$

$$\text{for } \bar{c}_p/c_{p_l} > 1 \quad Nu/Nu_0 = \left(\frac{\bar{c}_p}{c_{pl}}\frac{\lambda_w}{\lambda_l}\right)^{0.2}. \tag{18}$$

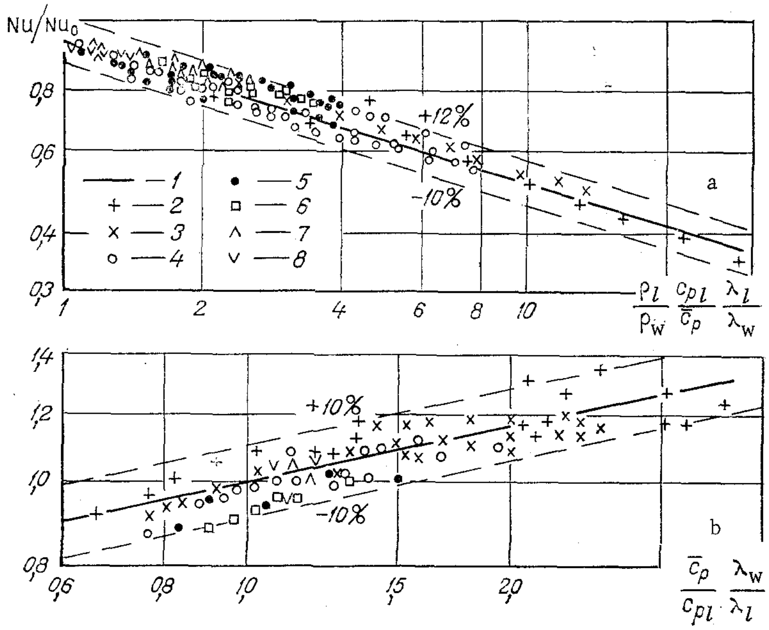

Fig. 4. Generalization of data on heat transfer: a) for $\bar{c}_p/c_{pl} < 1$; b) for $\bar{c}_p/c_{pl} > 1$; 1(a) – Eq. (17); 1(b) – Eq. (18); 2 – p = 0.25 MPa; 3 – 0.275; 4 – 0.3; 5 – 0.4; 6 – 0.6; 7 – 1.0; 8 – 2.0.

The calculations show that, at a prescribed pressure, the greatest reduction in Nu relative to Nu_0 occurs in the vicinity $T_l = T_m$. A relative increase in heat transfer is generally seen when $T_l < T_m$ and $\bar{c}_p/c_{pl} > 1$. Ignoring thermal accleration leads to some underestimation of heat transfer relative to correlations (17); (18), reaching about 10% at $\bar{c}_p/c_{pl} < 1$.

In conclusion, we should note that Eq. (17) also gives reasonable results for helium in the state of a perfect gas. In fact, assuming $\rho_l/\rho_w = T_w/T_l$, $c_{pl}/\bar{c}_p = 1$, and $\lambda_l/\lambda_w = (T_l/T_w)^{0.7}$, we obtain

$$Nu/Nu_0 = (T_w/T_l)^{-0.09}. \tag{19}$$

In the range $1 < T_w/T_l \leqslant 2$, this relation agrees within 10% with well-known calculated and empirical data [6-9].

NOTATION

u, axial velocity; $\bar{u} = \frac{2}{r_0^2}\int_0^{r_0} urdr$, mean velocity; $\overline{\rho u} = \frac{2}{r_0^2}\int_0^{r_0} \rho urdr$, mean mass velocity; h, enthalpy; h_l, mean mass enthalpy; T, temperature; T_l, mean mass temperature; T_m, pseudocritical temperature; p, pressure; p_c, critical pressure; x, coordinate along axis; r, running radius; r_0, tube radius; $Y = 1 - r/r_0$, dimensionless coordinate; τ, shear stress; q, heat flux; ρ, density; λ, thermal conductivity; μ, absolute viscosity; c_p, specific heat at constant volume; $\bar{c}_p = (h_w - h_l)/(T_w - T_l)$, mean specific heat; β, coefficient of cubical expansion; c_f, friction coefficient; $c_{f_0} = 16/Re$, friction coefficient for the case of constant properties; ξ_Σ, total-drag coefficient; Nu, Nusselt number; $Nu_0 = 4.36$, Nusselt number for the case of constant properties; Re, Reynolds number; Pr, Prandtl number; R, universal gas constant. Indices: w, wall; l, liquid; 0, constant pressure.

LITERATURE CITED

1. V. M. Yeroshenko and L. A. Yaskin, "Applicability of various correlations for the prediction of turbulent heat transfer of supercritical helium," Cryogenics, 21, No. 2, 94-96 (1981).
2. B. S. Petukhov and V. N. Popov, "Theoretical calculation of heat transfer and drag in laminar pipe flow of an incompressible liquid with variable properties," Teplofiz. Vys. Temp., 1, No. 2, 228-237 (1963).
3. V. N. Popov, "Calculating heat transfer and turbulent flow of a compressible liquid in a circular tube," Teplofiz. Vys. Temp., 15, No. 4, 795-801 (1977).

4. V. M. Eroshenko, L. I. Zaichik, and I. B. Zorin, "Calculation of drag and heat transfer in the laminar motion of an incompressible liquid with variable physical properties in a pipe with porous walls in the region of quasideveloped flow," Inzh.-Fiz. Zh., 39, No. 3, 462-467 (1980).
5. S. K. Vinokurov, "Study of hydrodynamics and heat transfer in the flow of a liquid with variable physical properties in channels with permeable walls," Heat and Mass Transfer in the Boiling and Flow of Cryogenic Liquids, A. V. Lykov Institute of Heat and Mass Transfer, Minsk (1980), pp. 80-93.
6. R. G. Deissler and A. F. Presler, "Analysis of developing laminar flow and heat transfer in a tube for a gas with variable properties," Proceedings of the 3rd Int. Heat Transfer Conference, Aug. 7-12, 1966, Chicago, Vol. 1, pp. 250-256.
7. P. M. Worsoe-Schmidt, "Heat transfer and friction for laminar flow of helium and carbon dioxide in a circular tube at high heating rate," Inst. J. Heat Mass Transfer, 9, No. 11, 1291-1295 (1966).
8. Davenport and Lepport, "Effect of temperature gradients on heat transfer and friction in a laminar gas flow," Teploperedacha, 87, No. 2, 36-43 (1965).
9. B. S. Petukhov, Heat Transfer and Drag in Laminar Flow in Pipes [in Russian], Énergiya, Moscow (1967).
10. R. D. McCarty, "Thermophysical properties of helium-4 from 2 to 1500°K with pressures to 1000 atm," NBS Technical Note 631 (1972).

HEAT-TRANSFER EQUATION FOR LAMINAR FLOW OF AROMATIC HYDROCARBONS AT SUPERCRITICAL PRESSURES

F. I. Kalbaliev, F. K. Babaev, and G. I. Isaev

UDC 536.242:62-987/-988

Experimental data on local heat-transfer coefficients for toluene and benzene ascending and descending in a vertical pipe is generalized in the form of criterional equations.

Determining the wall temperature of apparatus operating at supercritical pressures is one of the main problems of convective heat transfer, the solution of which has been the goal of numerous works on turbulent liquid flow. Heat transfer has been studied for laminar flow and supercritical pressure only for the aromatic hydrocarbons toluene and benzene [1-9] and polymethylphenylsiloxane liquid [10].

Tests on heat transfer involving aromatic hydrocarbons were conducted in a closed circuit on an experimental tube made of stainless steel with an inside diameter $d = 3$ mm, wall thickness $\delta = 0.5$ mm, and length of heated section $l = 200$-220 mm. The tests were conducted in the following parameter ranges:

for toluene

$$p/p_{cr} = 1.06 - 3.07,\ T_l/T_{cr} = 0.49 - 1.05,\ T_w/T_{cr} = 0.55 - 1.56,$$
$$q = (0.31 - 3.90)\cdot 10^5\ \mathrm{W/m^2},\ \mathrm{Re} = 375 - 4200;$$

for benzene

$$p/p_{cr} = 1.21 - 2.63,\ T_l/T_{cr} = 0.52 - 1.12,\ T_w/T_{cr} = 0.44 - 1.55,$$
$$q = (0.12 - 4.5)\cdot 10^5\ \mathrm{W/m^2},\ \mathrm{Re} = 320 - 5300.$$

Located ahead of the heated section of the tube is a hydrodynamic stabilization section of length $l_{h.s} = 0.06d\ \mathrm{Re}$. The length of the initial, thermal section, at $p > p_{cr}$, $T_l \ll T_m$, and $T_w \ll T_m$, is roughly $(1/\mathrm{Pe})(x/d) = 0.01$. At $p > p_{cr}$, $T_l < T_m$, and $T_w \gtrsim T_m$, the tempera-

M. Azizbekov Azerbaidzhan Institute of Petroleum and Chemistry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 3, pp. 379-383, March, 1983. Original article submitted July 6, 1981.

0022-0841/83/4403-0255$07.50